

Historical Open-High-Low-Close Volatility: Yang Zhang

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$$\sigma^2 = \sigma_o^2 + k\sigma_c^2 + (1 - k)\sigma_{rs}^2$$

$$\sigma_o^2 = \frac{Z}{n-1} \sum \left(\ln \frac{O_i}{C_{i-1}} - \mu_o \right)^2$$

$$\mu_o = \frac{1}{n} \sum \ln \frac{O_i}{C_{i-1}}$$

$$\sigma_c^2 = \frac{Z}{n-1} \sum \left(\ln \frac{C_i}{O_i} - \mu_c \right)^2$$

$$\mu_c = \frac{1}{n} \sum \ln \frac{C_i}{O_i}$$

$$\sigma_{rs}^2 = \frac{Z}{n} \sum \left(\ln \frac{H_i}{C_i} \ln \frac{H_i}{O_i} + \ln \frac{L_i}{C_i} \ln \frac{L_i}{O_i} \right)$$

$$k = \frac{0.34}{1 + \frac{n+1}{n-1}}$$

Yang and Zhang were the first to derive an historical volatility estimator that has a minimum estimation error, is independent of the drift, and independent of opening gaps. This estimator is maximally 14 times more efficient than the close-to-close estimator.

It can be interpreted as a weighted average of the Rogers and Satchell estimator, the close-open volatility and the open-close volatility.

The performance degrades to the classic close-to-close estimator when the price process is heavily dominated by opening jumps.

Symbol list:

σ	Volatility
Z	Number of closing prices in a year
n	Number of historical prices used for the volatility estimate
O_i	The opening price
H_i	The high
L_i	The low
C_i	The close