

Probability of the low of geometric Brownian motion

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$$P\left(\min_{0 < t < T} S_t \leq L\right) = \frac{1}{2} \operatorname{Erfc}(d_1) + \frac{1}{2} \left(\frac{L}{S_0}\right)^{\frac{2\mu}{\sigma^2} - 1} \operatorname{Erfc}(d_2)$$

$$d_1 = \frac{\ln \frac{S_0}{L} + (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{2t}}$$

$$d_2 = \frac{\ln \frac{S_0}{L} - (\mu - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{2t}}$$

$$\operatorname{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt$$

Cumulative probability function of the high of an underlying between now and some future time t. The underlying behavior is geometric Brownian motion.

Symbol list:

$P(x)$	Probability of x
L	Low level
S_0	Initial value at t=0 of the Brownian motion
μ	Drift term
σ	Volatility